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ON FORECASTING THE MOVEMENT OF LONG WAVES IN THE WESTERLIES AT THE 500 MILLIBAR LEVEL

by

W. J. ZINCK

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ON FORECASTING THE MOVEMENT OF LONG WAVES IN THE WESTERLIES AT THE 500 MILLIBAR LEVEL

by

William Joseph Zinck
Lieutenant (junior grade), United States Coast Guard

Submitted in partial fulfillment
of the requirements
for the degree of

MASTER OF SCIENCE IN AEROLOGY

U. S. Naval Postgraduate School
Monterey, California
1949

Library
U. S. Naval Postgraduate School
Monterey, California

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Department of Aerology

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14242

PREFACE

The purpose of this research was to investigate the utility of Rossby's wave equation in the westerlies at the 500 millibar level by statistical methods. Also a statistical analysis was made to investigate just how wave length, zonal index and latitude contribute to the movement of troughs in the westerlies.

This work was conducted at the United States Naval Postgraduate School, Monterey, California, during the period December 1948 to May 1949. It was done to meet partial requirements for the degree of Master of Science in Aerology. The idea for the investigation was suggested by Professor F. L. Martin of the staff of the Department of Aerological Engineering of the United States Naval Postgraduate School. His assistance and advice in the preparation of this thesis is gratefully acknowledged.

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TABLE OF

SYMBOLS AND ABBREVIATIONS

C_R - Eastward speed of trough at 500 millibars as computed by Rossby's wave equation.

C - Actual eastward speed of trough at 500 millibars.

U - Mean west to east wind component.

ΔZ - Height difference for 10° of latitude, 5° on each side of mean latitude.

β - Northward rate of change of Coriolis parometer.

L - Actual wave length, measured from the trough involved westward to the next major trough.

L_s - Stationary wave length.

ϕ - Latitude.

R - Mean radius of earth.

Ω - Angular velocity of earth.

σ - Standard deviation.

r - Correlation coefficient.

S - Standard error of estimate.

CHAPTER I

INTRODUCTION

The problem of forecasting the movement of long waves in the westerlies is very important in Meteorology. Although several approaches and techniques have been developed in an attempt to forecast the movement of upper air troughs, no perfect technique has been developed to accomplish this aim. As yet, no theoretical or statistical investigation has shown that the movement of these upper air troughs can be predicted with any great degree of accuracy.

The purpose of this paper is to make an objective investigation of the forecasting of the movement of major troughs in the westerlies at the 500 millibar level. The movement of these major troughs will be investigated in the zone 30 to 50 degrees north latitude, mainly over the continent of North America and adjacent oceans. The problem is complex and it will be approached in several ways, mainly statistical.

It will be of interest to consider previous approaches developed in an attempt to solve this problem. First, there is the purely kinematic approach of S. Petterssen (11). We will, however, concern ourselves primarily with the dynamical and statistical approach.

The main dynamical approach is that of Rossby (12) who developed the equation

$$C_R = U - \frac{\beta L^2}{4\pi^2} \qquad (1)$$

where

$$\beta = \frac{2\Omega \cos\phi}{R}$$

This equation gives the west-east velocity, C_R, of sinusoidal waves in the westerlies, where U is the speed of the westerlies (assumed constant),

β is the morthward rate of change of the Coriolis parometer and L is the wave length. In addition to the above-mentioned assumptions, Rossby also assumed a homogeneous incompressible atmosphere, zero horizontal divergence, and a scale of motion large enough so that the variation of the Coriolis parameter with latitude is of importance in determing wave dimensions.

The theory was extended by Haurwitz (6) to take into account the curvature of the earth and the finite lateral extent of the wave. Holmboe (8) obtained similar results for the more general barotropic atmosphere.

In 1944, J. Bjerknes and Holmboe (1) presented a theory of wave motion in a baroclinic atmosphere. The field of horizontal divergence is evaluated from the pressure pattern by means of gradient-wind relationships, and on this basis the following relation is established: If $\bar{u}(z)$ denotes the speed of the westerly current at any height z, u_c an increasing function of wave length and c the wave velocity, then

(2) $$c = \bar{u}(h) - u_c$$

where h is the height at which the mass divergence in the horizontal velocity field is zero.

Charney (3) considered the case of a baroclinic atmospheric model having a mean zonal wind in agreement with that in middle latitudes for the month of February. This model, however, has no horizontal wind shear. It is shown that Rossby's simple wave formula for the speed of a barotropic wave will apply to the baroclinic wave if the constant value of the zonal wind in the formula is the mean zonal wind averaged with respect to pressure from the top to the bottom of the baroclinic atmosphere. For the February model already mentioned, Charney finds that this mean zonal wind is that in the vicinity of the 600 mb. level. It has been pointed out by Holmboe that the formula for the baroclinic wave speed is strictly true only at the level of non-divergence in the atmosphere, which level, therefore, is in the vicinity of 600 mb.

Due to the fact that Rossby's original formula (1) was derived on the basis of sinusoidal waves in the westerlies, it suggests the importance of applying it to major atmospheric waves rather than minor waves which disappear at relatively low levels in the atmosphere. Namias and Clapp (10) used Rossby's formula for calculating the movement of five day mean troughs at 45 degrees latitude during the winter. The correlation between this calculated movement and the actual full week trough movement was .66 when cases of new trough development were eliminated. In contrast to this, the application by Brooks (2) and Yehle (14) to daily trough displacements yielded very poor results, but they apparently neglected to differentiate between major and minor waves. However, Craig subsequently obtained a much better correlation by using major waves. This was done by selecting as the wave length the distance between the trough concerned and the next one to the west having approximately the same depth. Possibly, this improved result was due to the fact that selection of the most pronounced troughs is comparable to smoothing pressure patterns by constructing five day means.

Finally, Cressman (4) made an important contribution to the statistical study of long wave movements in the westerlies. He expressed Rossby's formula in the form

$$C = \frac{\beta}{4\pi^2}\left(L_s^2 - L^2\right) \qquad (3)$$

where L_s is the wave length of a hypothetical stationary wave defined by the mean zonal wind, $U = \frac{\beta L_s^2}{4\pi^2}$. Then the trough velocity can be determined by either L_s or U. Cressman constructed for each of 120 individual cases a meridional profile of the zonal wind at 600 mb., the level of non-divergence. This enabled him to select the proper value of U at each latitude. The latitude to be used in β was arbitrarily selected as the latitude of the maximum west wind on the profile. The meridional profile of the zonal wind at 600 mb. was computed with the aid of the hydrostatic equation from meridional profiles of the zonal wind and the temperature gradient at 700 mb.

By testing the wave formula and using the above technique, Cressman found an absolute error of only two degrees of longitude per day. However, he selected for study those special synoptic situations when the major waves were quasistationary or were moving with a constant speed for a period of several days.

Cressman's method depends upon a laborious extrapolation process to the 600 mb. level and thus is not a practical tool. In order to reduce some of this labor, he attempts to determine L_s from the 700 mb. chart. This is done by computing constant vorticity trajectories centered on the 700 mb. jet stream. He found that a value of L_s determined by using a 700 mb. mean zonal wind U centered on the jet stream and averaged over 4° latitude on either side of the jet, gives good agreement with the value of L_s determined from the 600 mb. zonal wind profile. By this device, the problem of determining L_s is brought back by objective methods to the 700 mb. level.

The results obtained by Cressman using (3) together with the value of L_s obtained from the 700 mb. chart, does not give as favorable results as the method based on the 600 mb. chart. In fact, he states, "the data showed no simple relation between the speed of the wave and the difference between stationary and actual wave lengths." Thus, while the 600 mb. method of analysis gives fair results, it is too laborious, while methods based on the use of the 700 mb. chart are not accurate enough for normal verification.

Even though the level of non-divergence is between 610 and 570 mb., according to Charney, it was decided to test Rossby's formula at the 500 mb. level. While it is expected that the value of U derived from this chart will be too large, it is believed that it may be consistently too large, thus giving a good correlation to the desired correct value of U from the 600 mb. chart.

The data used in this investigation were obtained from Northern Hemisphere Historical Weather Maps (7) which are drawn for the surface and the 500 mb. level. These maps, published quite recently, are considered the best Northern Hemisphere charts available at this time. The charts for the months of February, March and April, 1946, were used for this particular paper.

Two statistical methods of attack are pursued, namely:

1. A strict test of the Rossby formula using 500 mb. data by comparing C_R with actual trough speed C.
2. A more general statistical analysis of the problem based on determining the 24-hour trough velocity C as a linear regression on the variates, wave length L, the mean latitude ϕ, and the height difference Δz.

The solution of problems 1 and 2 necessitates objective methods of determining the variates. These objective methods are described in the following chapter.

CHAPTER II

METHOD OF ANALYSIS

This chapter will describe in detail the method of analysis used in this investigation. It will consist mainly of the procedure used in obtaining the variates from the synoptic charts at the 500 mb. level. This procedure will also be the basis for using graphs based on this method of analysis.

Minor waves have their greatest amplitude in the low levels and only very small amplitudes at the height of the tropopause, and in general move very rapidly. However, major waves may be almost obscure at sea level, but the amplitude of these waves increases with height and reach their greatest strength at the tropopause. In this investigation, only major waves will be used, a major wave being defined as one which has a well-defined ridge lying between two adjacent troughs.

The wave length of these major waves is almost always so large, that if one trough is over the dense network of observations of North America, the other troughs are over the Pacific or Atlantic Ocean. Due to this fact, it is usually impossible to locate an upper air trough closer than to within five degrees of longitude. If this is true, it is reasonable to assume that the expected error in determining the actual wave length would be about seven percent if an average wave length of 70 degrees longitude is assumed.

Troughs were considered to be at the point of maximum curvature of the contour lines, but where the curvature was constant, the southernmost point of the contour line was used.

For utility, the wave formula may be given in a more convenient notation, following the convention that velocities are to be measured in degrees of longitude over which the wind or pertubation moves in a period of twenty-four hours and the wave length of the pertubation is measured in degrees of longitude rather than linear measure. Thus equation (1) becomes

$$C_R = U - \frac{\cos^2\phi}{180} L^2 \tag{4}$$

where ϕ is the latitude.

The objective methods of selecting variates are as follows:

1. Mean latitude of the wave. A contour line was selected which crossed the trough involved and the next trough to the west of it. This contour line was imbedded in the mean westerly current of the trough involved. Such a contour line is shown in Figure 1 as YY' The intersection of this contour line with the trough lines gave latitudes ϕ_1, and ϕ_2. The average of ϕ_1, and ϕ_2 gave the mean latitude. In this investigation, this mean latitude was rounded off to the nearest five degrees, probably without causing any error greater than observational errors.
2. The wave length was measured as the distance in degrees longitude from the trough under investigation westward to the next trough at the mean latitude.
3. The mean zonal wind was measured in a five degree zone both sides of the mean latitude and bounded by the trough involved and the next trough to the west.
4. Modification of Rossby's formula to constant pressure analysis.

$$C_R = \frac{2.86\,\Delta Z}{\sin 2\phi} - \frac{\cos^2\phi}{180} L^2 \tag{5}$$

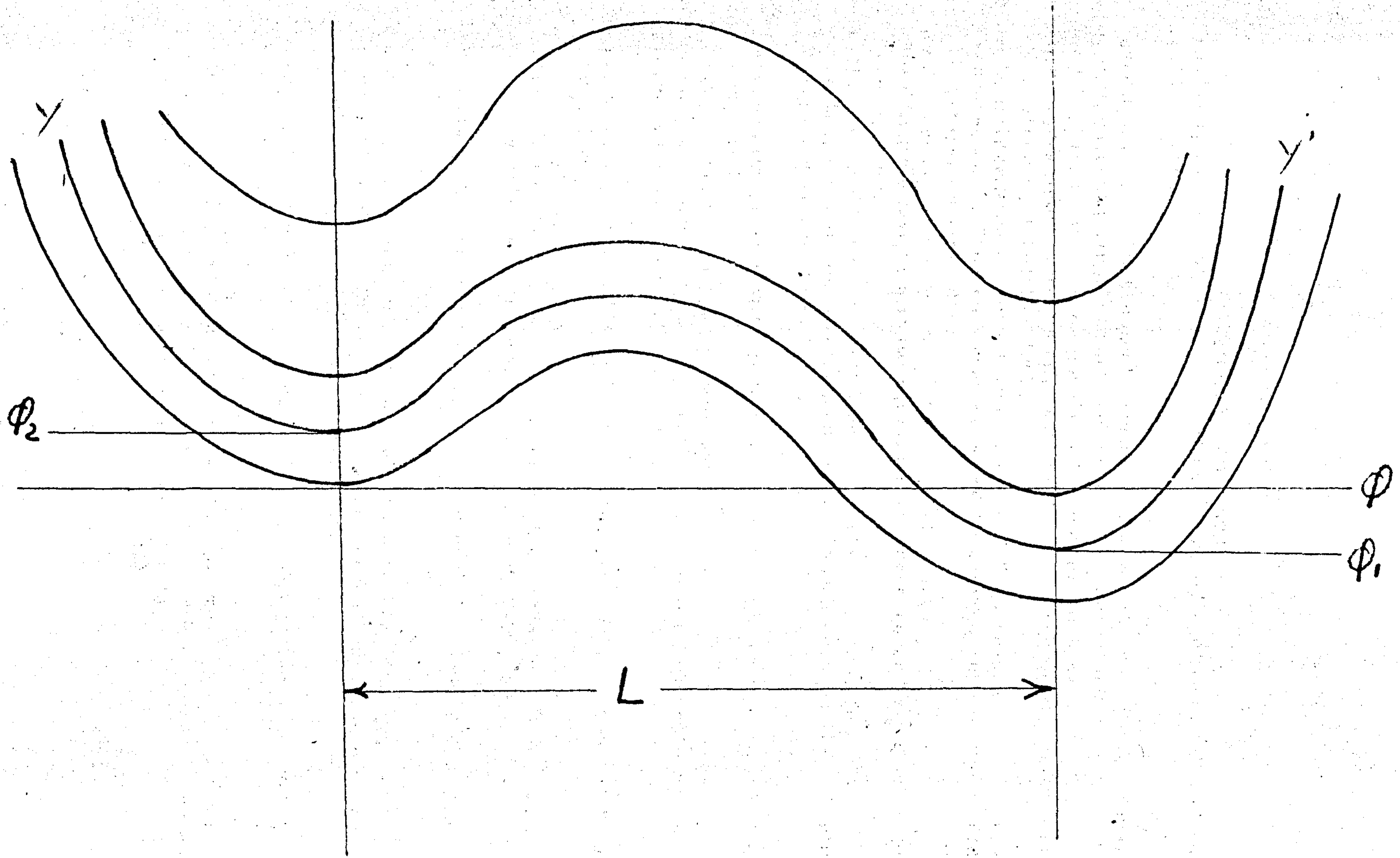


Fig. 1 (a)

ΔZ is the average height difference per ten degrees of latitude centered on the mean latitude. This modification is based upon the assumption that the zonal wind is the geostrophic wind.

In selecting synoptic situations, there were only two limitations. First, there must be a broad band of westerlies which did not contain any closed centers, since the east winds associated with the closed centers would tend to cancel out an equal amount of west winds in averaging out the height differences so that the final zonal wind profile would not give an accurate representation of the strength of the westerlies. Secondly, only waves which approached sinusoidal curves, and did not have exceptionally large or small amplitudes were used.

In all, 88 cases satisfying these criteria were selected during the months of February, March and April, 1946, with a mean latitude range of 30 to 50 degrees north latitude.

CHAPTER III

RESULTS AND INTERPRETATION

All values of speed shown in these results will be in units of degrees of longitude per day. The results of a test of Rossby's formula follow:

$$\bar{C} = 10.4 \pm 4.1 \quad *$$

$$\bar{C}_R = 11.9 \pm 5.2 \quad *$$

The linear regression of C on C is given by

$$C = .39C_R + 5.78 \tag{6}$$

with a correlation coefficient between C and C_R of .49. The standard error of estimate in estimating C from C_R was 3.6° longitude per day.

From the mean values shown above, it is seen that the wave formula tends on the average to overestimate the speed of the troughs by a degree and a half. This is not surprising, since strictly speaking, the formula is valid only in the vicinity of the non-divergence level, approximately 600 mbs. However, from the regression equation, C and C_R are equal when they have a value of 9.46 degrees longitude per day. Therefore, the conclusion already drawn from the arithmetic means alone is only true when C_R is greater than 9.46, which is the case in the majority of observations. When C_R is less than 9.46, C is greater than the calculated C_R.

Now we will correlate the variates C on Δz, L and φ. The units are as follows: C in degrees longitude per day, Δz in hundreds of feet, L in degrees longitude, and φ in radians. The simple correlations are shown in

---

* Standard deviation was used here. See Kenney (9), p. 114.

Table II. The results of the multi-variate analysis follow:

Multiple correlation coefficient

$$r(C \cdot \Delta z, L, \phi) = .63$$

Standard error of estimate

$$S(C \cdot \Delta z, L, \phi) = 3.13^{\circ} \text{ Longitude/Day}$$

Linear regression equation

$$C = .16 \Delta z - .12 L + 19.96 \phi + 1.08 \tag{7}$$

In a test of regression equation (7) the following distribution of computed minus observed displacements are shown here. The units are degrees longitude per day.

$< -6^{\circ}$	-6° to -2°	-2° to $+2^{\circ}$	$+2^{\circ}$ to $+6^{\circ}$	$> 6^{\circ}$
3	12	55	16	2

It is seen the regression equation (7) gives a normal distribution with the errors distributed almost equally on each side of the central block. In 62.5% of the cases, the difference is less than 2 degrees longitude and 94% of the cases show a difference of less than 6 degrees.

The average absolute errors as given by regression equations (6) and (7) are 2.76 and 2.46 degrees longitude per day, respectively.

In interpreting the results, it is seen that the simple correlation coefficients agree qualitatively with Rossby's wave formula (1). The correlation coefficient of +.34 between the observed speed of the trough and the height difference indicates that the speed of the trough varies directly with the zonal index. Also the correlation coefficient of -.30 between the speed

TABLE I

		C	ΔZ	L	ϕ
Means of Variates		10.41	6.38	45.42	.70
Standard Deviation		4.1	1.4	11.2	.10
Simple Correlation Coefficients	C	—	.34	-.30	.53
	ΔZ	.34	—	.05	.58
	L	-.30	.05	—	.06
	ϕ	.53	.58	.06	—

of the trough and the wave length shows that the speed of the trough decreases as wave length increases.

Probably of greater significance is the fact that a correlation coefficient of +.53 was found between the speed of the trough and the latitude. This effect of latitude agrees quite well with Rossby's wave formula (1) where the value of β decreases by about 26% from 30 to 60 degrees latitude. Although the latitude has an effect on both terms of equation (5), its effect on the last term of the equation is more predominant, which agrees well with the results.

The simple correlation coefficient of +.58 between the latitude and the height difference is due to the closed highs at low latitudes as shown by the synoptic charts. These closed highs cause a low zonal index which is shown in the results.

Although the simple correlation coefficients agree quite well with Rossby's wave formula (1) qualitatively, they do not agree quantitatively. This is shown by the fact that the correlation coefficient between C and C_R is only .49, with a standard error of estimate for C of 3.6° longitude per day.

By developing the linear regression equation (7), the correlation coefficient was raised to .63. While this is considerably better than .49, it does not lower the standard error of estimate appreciably. However, equation (7) does help to show quantitatively the effect of height difference (Δz), wave length (L), and latitude (ϕ) on the speed of the trough (C). For an average case, the latitude effect is about 12 times as important as the height difference term and about twice as important as the wave length term. This would indicate that at a specific latitude, the movement of the trough would depend primarily on the wave length and that the zonal index plays a very small part. In fact, the zonal index term would have a maximum value of 1.5° longitude per day for the average case. With present methods of obtaining meteorological information, the movement of a trough at 500 mb. could not be measured to 1 or 1.5 degrees of longitude, so the zonal index term may be considered negligible as compared to the wave length and latitude terms. These results agree quite well with those of Namias and Clapp (10).

As the development of a tool for the forecasting of movement of troughs at the 500 mb. was one of the primary objectives of this paper, a graph is plotted which gives trough velocity when the mean latitude, height difference and wave length are known. However, it is necessary to use the technique of analysis as described in Chapter II. The main conclusions are stated in the following:

SUMMARY

1. Rossby's wave equation (1) is not applicable at the 500 mb. level.
2. Equation (6), when used to estimate trough speed, gives a standard error of estimate of 3.6° longitude per day with an average absolute error of 2.76° longitude per day.
3. Equation (7), when used to estimate trough speed, gives a standard error of estimate of 3.13° longitude per day with an average absolute error of 2.46° longitude per day.
4. The zonal index plays a relatively small part in the movement of troughs at the 500 mb. level. This is further substantiated by an analysis of the synoptic 500 mb. chart. In every case where there was a closed high behind the trough, the observed movement was greater than the calculated movement. This indicates that the trough moved even though there was a very weak zonal flow from west to east.

5. The fact that the movement of the trough as calculated by Rossby's wave equation (1) overestimates the actual trough movement indicates that the level of non-divergence is below 500 mb.
6. Even after considering the errors introduced due to present observational techniques, it is apparent that there must be another variate which contributes to the speed of the trough. It is quite possible that this variate is wave amplitude.

REGRESSION OF "C" ON "C_R"

UNITS - DEGREES LONGITUDE / DAY

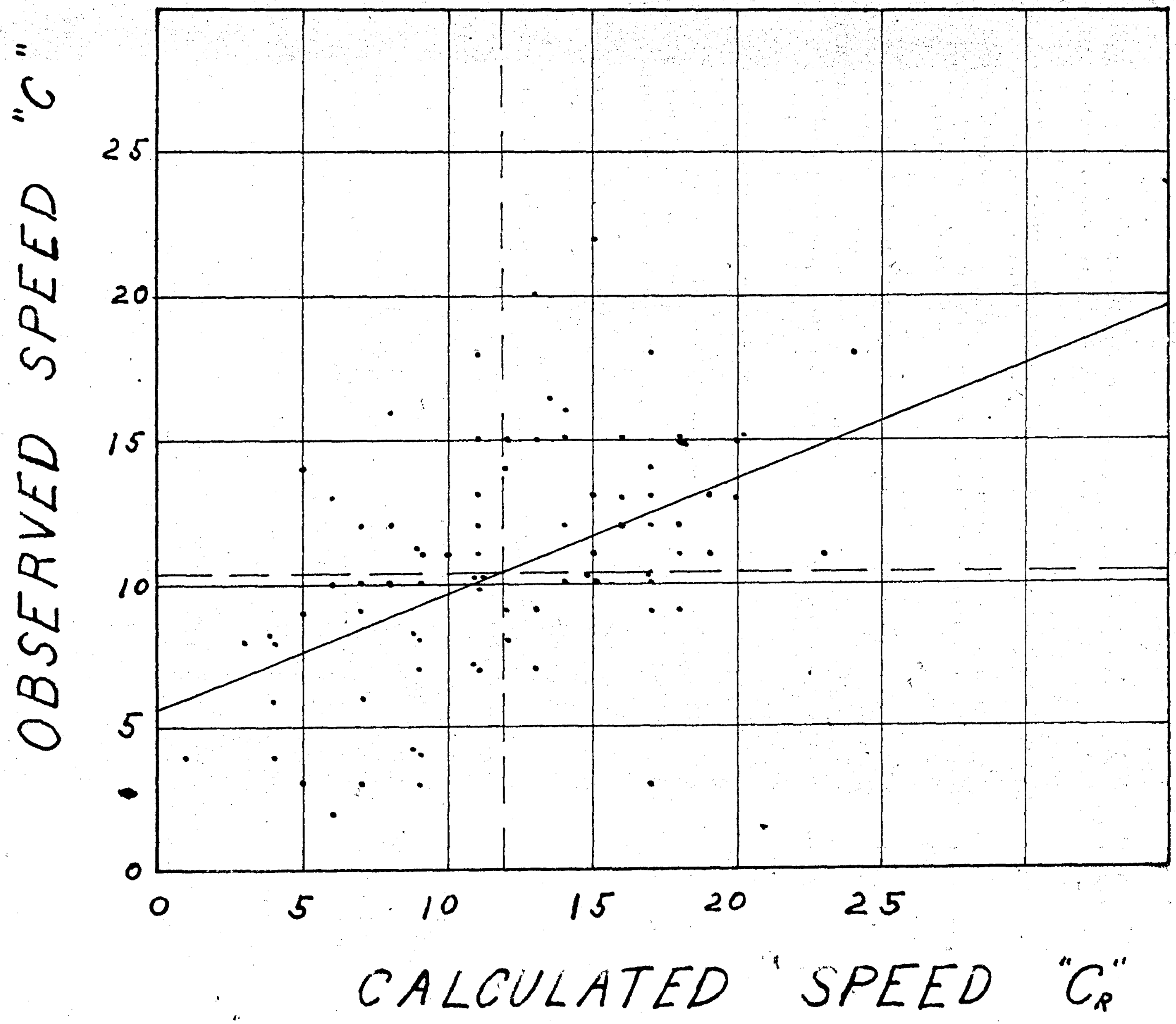
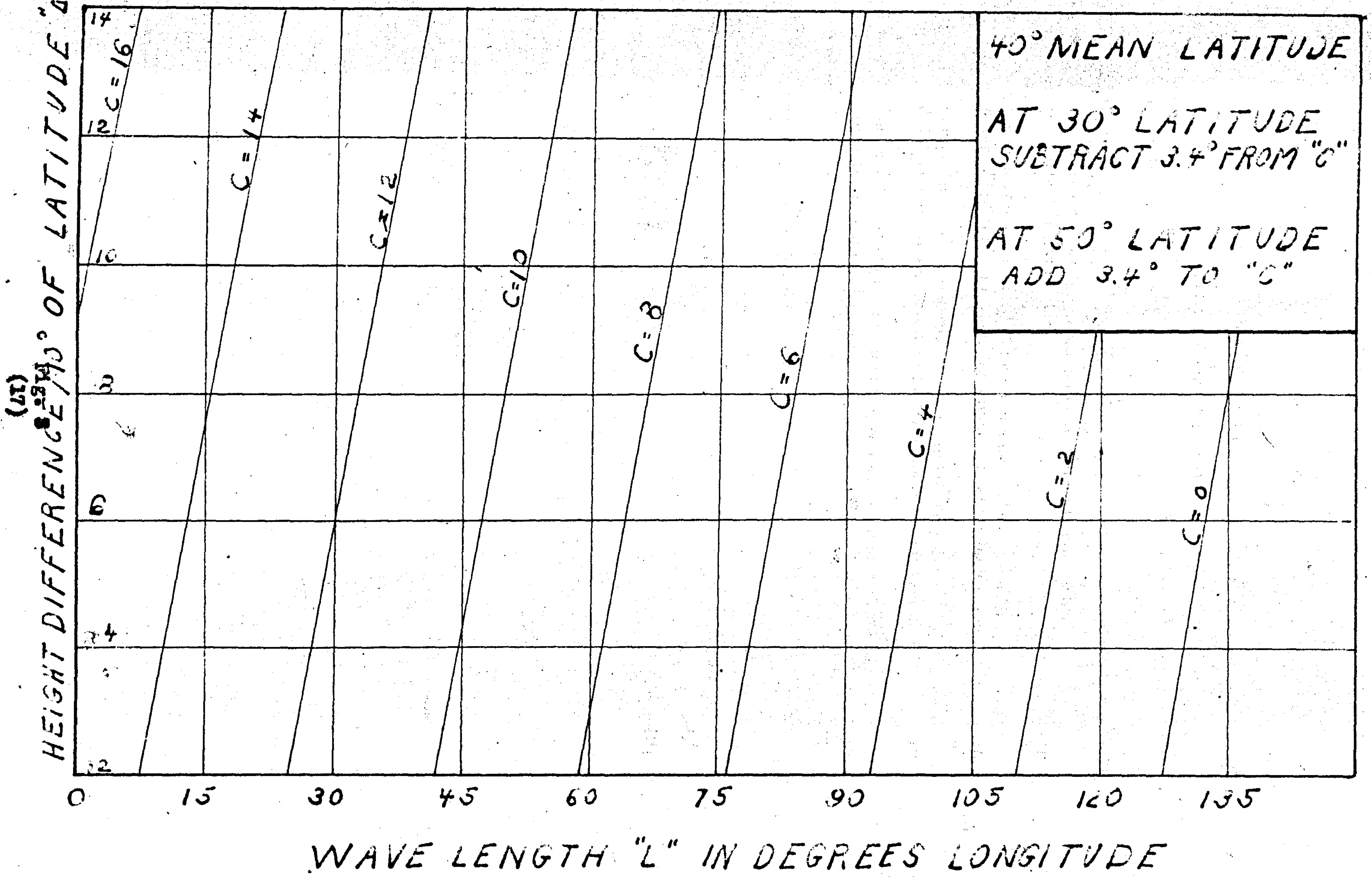


Fig. 2

LINES OF CONSTANT SPEED "C" IN DEGREES LONGITUDE/DAY



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APPENDIX I

VARIATION OF β WITH LATITUDE

90°	0.0
75°	0.593
60°	1.145
50°	1.461
45°	1.619
30°	1.983
15°	2.212
0°	2.290